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SCALAR DILATON-QUARKONIUM MESON IN NUCLEON STRUCTURE

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Static properties and electromagnetic form factors of nucleons are calculated in the *Generalized Skyrme model* with an explicit scalar dilaton-quarkonium meson which saturates the quark-loop contribution to the scale anomaly of QCD.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Скалярный мезон-дилатон-кварконий в нуклонной структуре

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Вычислены электромагнитные формфакторы и статические свойства нуклонов в обобщенной модели Скирма, включающей скалярное поле дилатона-кваркония, которое насыщает вклад кварковой петли в масштабную аномалию КХД.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

The main features of quantum chromodynamics (QCD) at low energies are the spontaneous breaking symmetry via Nambu-Jona-Lasinio mechanism and breaking of the scale symmetry so that the divergence of the dilatational current does not vanish[1]. The QCD low-energy region is governed by quark and gluon fluctuations leading to a formation of the non-zero values of the quark $\langle\bar{\psi}\psi\rangle$ and gluon $\left\langle \frac{\alpha_s}{\pi}\left(G^a_{\mu\nu}\right)^2\right\rangle$ condensates. For the large number of color N_c one can consider QCD as an effective theory of mesons only[2]. As for baryons one can follow the Skyrme

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idea that the baryon can emerge as a soliton in the chiral meson theory[3]. Considerations of the Skyrme model showed[4],[5] that we can describe satisfactorily the basic static properties of the nucleon except for the masses which are too large. One can try to include the vector mesons (ω, ρ, a_1) to the non-linear sigma model to stabilize soliton [6],[7], but the problem of large masses remains. But this approach does not take into account the scale anomaly of the QCD and therefore omits scalar particles which are essential for understanding the intermediate range attraction in the nucleon-nucleon interaction[8], the nature of baryon resonances[7], and reduction of the classical mass component.

At present time there are two different approaches to include scalar meson-dilaton into the Skyrme model. In both approaches the interaction of the dilaton field with the chiral field is dictated by the scale invariance. The main difference is in the origin of the dilaton. On one hand, there is the approach in which dilaton is associated with the glueball[9]. In this approach the glueball field saturates the scale anomaly completely. The potential[10] which reproduces for pure gluodynamics the scale anomaly and QCD scalar sum rules is considered. In the other approach, the dilaton is treated as a quarkonium arising due to fluctuations of the quark condensate[11],[12], which is an order parameter for the chiral symmetry breaking. In this approach the dilaton-quarkonium saturates a part of the scale anomaly, namely the part which is produced by the quark loops. The choice of the quarkonium as a dilaton is based on two principal observations:
a) The experimental studies of scalar resonances [13] show that the real lightest candidate for the glueball state is $f_0(1590)$ (former $G(1590)$) [14] which does not appear in $\pi\pi$ and $K\bar{K}$ productions.
b) Consideration of the chiral anomaly[15] shows that the only gauge-invariant combination of the gluon field $G_{\mu\nu}G_{\lambda\rho}$ can contact only with a total antisymmetric tensor $\varepsilon^{\mu\nu\lambda\rho}$. Thus this combination has $J^{PC} = 0^{-+}$ quantum numbers and contributes to Wess-Zumino-Witten action producing $U(1)$ anomaly.

It suggests that the 0^{++} glueball field as a fluctuation of $G^2_{\mu\nu}$ cannot interact directly with the chiral field, but only through the mixing with the dilaton-quarkonium. The estimate gives small value for the mixing angle of glueball and quarkonium states.

Our model is based on the effective low-energy action for pseudoscalar and scalar (dilaton-quarkonium) mesons which was derived in[16] starting directly from the QCD generating functional by the joint chiral and conformal bosonization method. This lagrangian favours the linear sigma-model in terms of the composite field $\phi = F_\pi exp(-\sigma)U$. In this paper we present the calculations of the main properties of the nucleon in the two-flavor model, which is based on this effective action. We use the notation of Ref.[16].

We define the effective action $W_{eff}(U,\sigma)$ for chiral and scalar (dilaton- quarkonium) fields by

$$Z_\psi(\not{D})Z_{inv}^{-1}(\not{D}) = \int_L D\Phi exp[-W_{eff}(U,\sigma)] \ ,$$
$$\Phi = e^{\sigma(x)/2}U^{1/2}(x), \tag{1}$$

where the functional

$$Z_{inv}^{-1}(\not{D}) = \int_L D\Phi Z^{-1}(\Phi \not{D} \Phi) \tag{2}$$

is invariant under local and conformal transformations of quark fields and should be approximately constant in low energy region L. The effective action $W_{eff}(U,\sigma)$ can be expressed using the diagonal part of the projection operator onto the subspace $||\not{D} - M|| \leq \Lambda$ of the eigenvalues k. One can calculate the diagonal part of the projector using the finite mode regularisation [17] of the functional integral. The relevant expression for the effective Lagrangian for the dilaton- quarkonium and chiral fields at low energy can be written in the form of generalized linear σ model for the field $\phi = F_\pi exp(-\sigma)U$ [16]:

$$L_{eff}(\phi) = \frac{1}{4}Tr\left(\partial_\mu\phi^+\partial^\mu\phi\right) - V(\phi) + L^{(4)}(\phi) \ , \tag{3}$$

$$V(\phi) = Tr\left\{\frac{C_g}{48F_\pi^4}\left(\phi^+\phi\right)^2 + \frac{m_\pi^2}{4F_\pi^2}\phi^+\phi\left(\phi^+ + \phi\right) - \right.$$
$$\left. - \frac{N_f}{24\pi^2}Tr_c G_{\mu\nu}^2 ln\left(\frac{\phi^+\phi}{F_\pi^2}\right)\right\} . \tag{4}$$

The last two terms exactly reflect a contribution of a quark loop to the scale anomaly, i.e., nonvanishing divergence of the dilatational current D_μ

$$\partial_\mu D^\mu = T^\mu_\mu = \frac{\beta(g)}{g} G^2_{\mu\nu} + \sum_i (1+\gamma_i) m_i \bar{\psi}_i \psi_i \tag{5}$$

in the lover energy region L with zero values of anomalous dimensions γ_i. The most interesting is the last term in (4) proportional to $G^2_{\mu\nu}$ which corresponds to the contribution of quarks to the Gell-Mann - Low β-function, which in one-loop approximation contains pure gluonic part (proportional to the number of colors N_c) and fermionic part (proportional to the numbers of flavors N_f)

$$\beta(g) = -\frac{\frac{11}{3}N_c - \frac{2}{3}N_f}{16\pi^2} g^3 \tag{6}$$

From the other hand, this term determines the mixing between the dilaton-quarkonium and the colorless configuration $Tr_c G^2_{\mu\nu}$ of the gluon field (glueball). The equation (4) suggests that the glueball field cannot interact directly with the chiral field, but only through the coupling with the quarkonium-dilaton meson. As the next step, one can introduce one-loop potential for glueball field[10], diagonalize the mass matrix and consider the generalization of Skyrme model with two 0^{++} scalar meson-glueball and quarkonium. We estimate a mixing angle Θ between the glueball G(1590) and quarkonium fields using Eqs. (4) and (5) in the spirit of the Ref.[18] and find that does non exceed 20. The small value of Θ is consistent with the QCD sum rules approach. Thus, it is a good approximation to consider only the contribution of dilaton-quarkonium to the formation of the baryon as a chiral soliton.

As one cannot deduce the effective Lagrangian for the glueball fields from the low-energy QCD we prefer to integrate out the gluon fields with some plausible assumptions. The full generating functional

$$Z_L = \int DG \, exp\{iW_{YM}\} \int D\Phi \, exp\{iW_{eff}(\sigma, G)\} \tag{7}$$

includes the integration over the gluon fields with the weight $exp[iW_{YM}]$. We combine the last term in (4) with the effective Yang-Mills Lagrangian and find that a variation of the averaged value of the dilaton field $\langle\sigma_0\rangle = \sigma_c$ changes the effective coupling constant

$$\left(\frac{1}{2g^2} + \frac{N_f\sigma_c}{24\pi^2}\right) Tr_c G^2_{\mu\nu} = \frac{1}{2g^2_{eff}} Tr_c G^2_{\mu\nu} \ . \tag{8}$$

At large positive values of σ_c the effective coupling constant g_{eff} decreases, going over to the regime of the asymptotic freedom. This obstacle enables us to calculate the effective potential not only in the region of low energies but also in the perturbative region. Due to the asymptotic freedom, one can solve the renormalization group equation for the vacuum energy in one-loop approximation for the Gell-Mann-Low β-function [10] $\beta \simeq -bg^3$, $b = -(33 - 2N_f)/48\pi^2$ using the correlation between the vacuum energy and the trace of the energy-momentum tensor

$$E_{vac} = \frac{1}{4}\left\langle T^\mu_\mu \right\rangle \ , \ T^\mu_\mu = \frac{\beta(g)}{2g}\left(G^a_{\mu\nu}\right)^2 \tag{9}$$

and find the principal contribution to the effective potential in the asymptotic freedom region (large σ_c)[15]:

$$V_G(\sigma_0) = -\left\langle \frac{\alpha_s}{\pi}\left(G^a_{\mu\nu}\right)^2 \right\rangle \frac{11N_c - 2N_f}{32N_c} e^{\varepsilon\sigma_0} \ , \ \varepsilon = \frac{N_f}{6\pi^2 b} \tag{10}$$

which replaces the last term in Eq.(4).

The corresponding effective lagrangian for the chiral and scalar fields in the limits of the large N_c is

$$\begin{aligned} L_{eff}(U,\sigma) &= \frac{F^2_\pi}{4} e^{-2\sigma} Tr\left[\partial_\mu U \partial^\mu U^+\right] + \frac{N_f F^2_\pi}{4}\left(\partial_\mu\sigma\right)^2 e^{-2\sigma} + \\ &+ \frac{1}{128\pi^2} Tr\left[\partial_\mu U U^+, \partial_\nu U U^+\right]^2 - \\ &- \frac{C_g N_f}{48}\left(e^{-4\sigma} - 1 + \frac{4}{\varepsilon}(1 - e^{-\varepsilon\sigma})\right) . \end{aligned} \tag{11}$$

This lagrangian is a generalization of the well-known Skyrme model[3] and takes into account the conformal anomaly of the

QCD. The first two terms are the kinetic terms of the chiral and scalar fields. The kinetic term of the chiral field has an additional scale factor $exp(-2\sigma)$ in comparison with the Skyrme model. The third term is a well-known Skyrme term. The effective potential for the scalar field is a result of the extrapolation of the low-energy potential into the high energy region in one-loop approximation to the Gell-Mann-Low QCD β-function. The parameter ε depends on the number of flavors N_f: $\varepsilon = 8N_f/(33-2N_f)$.

In the baryonic sector we choose for the chiral field the spherically symmetrical static Skyrme ansatz $U(\vec{x}) = exp[i\tau \cdot \hat{n}F(r)]$ where $\hat{n} = \vec{r}/|\vec{r}|$ and τ is Pauli matrixes. It is convenient to introduce a new field $\rho(x) = exp(-\sigma(x))$. The mass functional for the dimensionless variable $x = eF_\pi r$ has the form $M = M_2 + M_4 + V$, where

$$M_2 = 4\pi\frac{F_\pi}{e}\int\limits_0^{+\infty} dx\left[\frac{N_f}{4}x^2(\rho')^2 + \rho^2\left(\frac{x^2(F')^2}{2} + sin^2 F\right)\right] \ , \quad (12)$$

$$M_4 = 4\pi\frac{F_\pi}{e}\int\limits_0^{+\infty} dx\left[\frac{sin^2 F}{2x^2} + (F')^2\right] sin^2 F \ , \quad (13)$$

$$V = 4\pi\frac{F_\pi}{e}D_{eff}\int\limits_0^{+\infty} dx x^2\left[\rho^4 - 1 + \frac{4}{\varepsilon}\cdot(1-\rho^\varepsilon)\right] \ . \quad (14)$$

In Eqs.(12-14) the Skyrme parameter e is equal to 2π in accordance with Eq.(11). For the factor D_{eff} we have $D_{eff} = C_g N_f/48e^2F_\pi^4$. For the pion decay constant F_π we take its experimental value $F_\pi = 93MeV$. The value of the gluon condensate is estimated by QCD sum rules $C_g = (300-400MeV)^4$. The Euler-Lagrange equations for the shape functions $F(x)$ and $\rho(x)$ follow to be:

$$F''\left[\rho^2 x^2 + 2sin^2 F\right] + 2F'x\left[x\rho\rho' + \rho^2\right] + F'sin(2F) -$$

$$-\rho^2 sin(2F) - sin(2F)sin^2 F/x^2 = 0 \ , \quad (15)$$

$$\frac{N_f}{2}x\left[x\rho'' + 2\rho'\right] - 2\rho\left[\frac{x^2(F')^2}{2} + sin^2 F\right] -$$
$$-4D_{eff}\left[\rho^3 - \rho^{\varepsilon-1}\right] = 0, \tag{16}$$

where prime corresponds to the derivative with respect to x. The solutions of the Eqs.(15-16) are graphically represented in Figs.1 and 2. According to the virial theorem, the contributions of

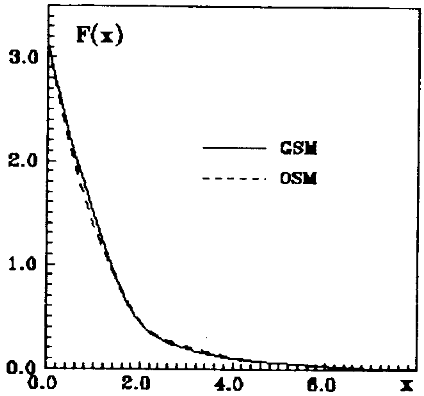

Fig.1. Chiral angle $F(x)$ in GSM and OSM for $C_g = (300\text{MeV})^4$ and $N_f = 2$.

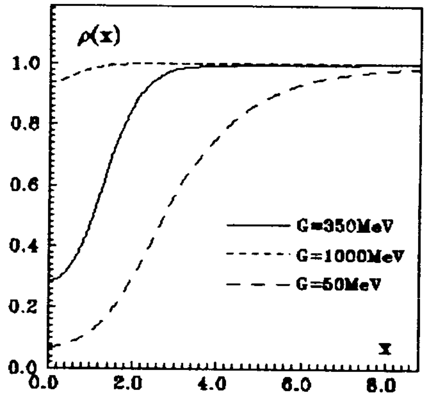

Fig.2. Scalar meson shape function for $G = C_g^{1/4} = 50$ Mev, 350 MeV and 1.0 GeV.

the individual terms of the functional on solutions of the system must satisfy the condition $M_2 - M_4 - 3V = 0$, which can be used to control accuracy of the numerical solution of the system. The asymptotic behaviour at large distances for $F(x)$ is identical to the Skyrme model $F(x) \sim a/x^2$ and behaviour of $\rho(x)$ exhibits a rapid downfall from the unity: $\rho(x) \sim 1 - b/x^6 + \dots$. The investigation of Eq.(15-16) at small distances gives: $F \sim \pi N - \alpha x$, $\rho \sim \rho(0) + \beta x^2$, $\rho(0) \neq 0$. The boundary conditions ensure a finite mass functional for a given value of the topological charge N.

Note, that there is another solution when the function $\rho(x)$ vanishes at the finite value of radius $x = x_c$, $x_c \neq 0$. Due to the factor $exp(-2\sigma)$ in front of the kinetic chiral term in (8) the dynamical chiral field does not propagate at distances less than x_c and the bag-like structure emerges.The baryon charge is quantized on non-topological grounds in order to have the mass functional finite.

In this paper we look for nonvanishing solutions for $\rho(x)$ at origin and the chiral shape function $F(x)$ has the same boundary conditions in the main order as in the original Skyrme model (OSM). That is the reason why we call this model: *generalized Skyrme model* (GSM). To calculate the properties of the baryon in GSM we introduce breather and rotational degrees of freedom[5] as a collective coordinates. We choose the time-dependent chiral and scalar fields in the form

$$U(\vec{r},t) = A(t)U_0\left(e^{\lambda}(t)\vec{r}\right)A^{+}(t)\ , \quad \rho(\vec{r},t) = \rho_0\left(e^{\lambda}(t)\vec{r}\right)\ . \tag{17}$$

The time-dependent scalar parameter λ plays the role of the collective variable describing breather vibrations of the solutions of the stationary equations $U_0(\vec{r})$ and $\rho(\vec{r})$. After canonical quantization and diagonalization in angular variables[5], we obtain the effective Hamiltonian

$$\hat{H} = \frac{\hat{P}_\lambda^2}{2m(\lambda)} + M(\lambda) + \frac{\hat{S}^2}{2I(\lambda)}\ . \tag{18}$$

Here $\hat{P}_\lambda$ is the quantum momentum operator corresponding to vibrations and $\hat{S}$ is the operator of spin. The effective mass $m(\lambda)$, the potential $M(\lambda)$ and the moment of inertia $I(\lambda)$ are given by the expression

$$m(\lambda) = e^{-3\lambda}Q_2 + e^{-\lambda}Q_4\ , \tag{19}$$

$$M(\lambda) = e^{-\lambda}M_2 + e^{\lambda}M_4 + e^{-3\lambda}V\ , \tag{20}$$

$$I(\lambda) = e^{-3\lambda}I_2 + e^{-\lambda}I_4\ . \tag{21}$$

The coefficients M_2, M_4 and V are given by Eqs.(12-14). Q_2, Q_4, I_2 and I_4 are the values of the following integrals:

$$Q_2 = \frac{\pi}{e^3F_\pi}\int_0^\infty dx x^4\left[\rho^2(F')^2 + \frac{N_f}{4}(\rho')^2\right]\ , \tag{22}$$

$$Q_4 = \frac{8\pi}{e^3F_\pi}\int_0^\infty dx x^2(F')^2 sin^2F\ , \tag{23}$$

$$I_2 = \frac{4\pi}{3}\frac{1}{F_\pi e^3}\int_0^\infty dx x^2 \rho^2 sin^2 F \ , \tag{24}$$

$$I_4 = \frac{16\pi}{3}\frac{1}{F_\pi e^3}\int_0^\infty dx x^2 \left[(F')^2 - \frac{sin^2 F}{x^2}\right] sin^2 F \ . \tag{25}$$

Some numerical results are given in the Table and Fig.3, where the mean square root radius $\langle r_B^2 \rangle^{1/2}$ of the baryon charge

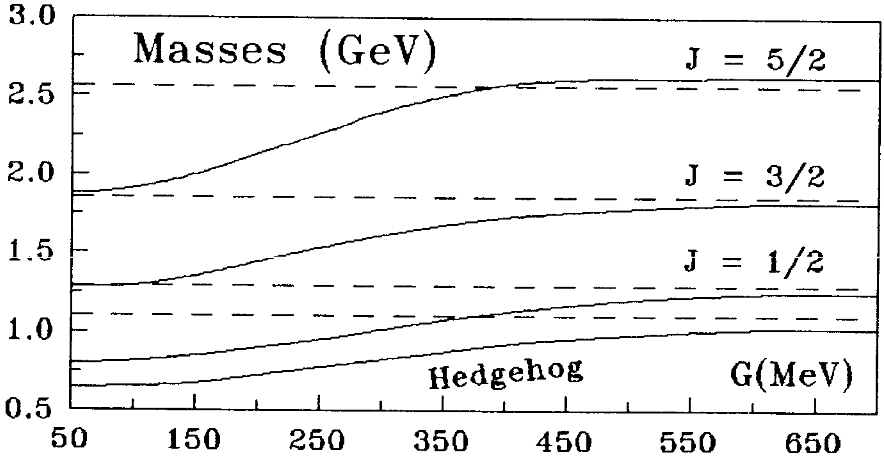

Fig.3. Mass spectra of the ground and excited states in GSM (solid line) and OSM (dashed line).

distribution

$$J_B(x) = -\frac{1}{F_\pi e}\frac{1}{2\pi^2}F'\frac{sin^2 F}{x^2} \tag{26}$$

and mean square root radius $< r_{iv}^2 >^{1/2}$ of the isovector charge distribution

$$J_{iv}(x) = sin^2 F \left[x^2\rho^2 + (F')^2 x^2 + sin^2 F\right] \tag{27}$$

are also shown.

The form factors of the neutron and proton have been calculated using the Eqs.(26-27) and graphically represented in Figs.4 and 5.

Table. Main static properties of solution with $B = 1$ in the two flavor *Generalized Skyrme Model* for the choice of the parameters $F_\pi = 93MeV$, $e = 2\pi$, $C_g = (300MeV)^4$. The results obtained in the *Original Skyrme Model* are given for the comparison.

Values	GSM without vibrations	GSM with vibrations	OSM without vibrations	OSM with vibrations
M_{cl}	839		1098	
M_n	1084	1026	1310	1288
$< r^2 >_{is}^{1/2}$	0.38	0.45	0.34	0.45
$< r^2 >_{iv}^{1/2}$	0.65	0.77	0.68	0.85

We should like to point out that the classical and rotational components of the baryon mass are much smaller as compared to the original Skyrme model. One can see a partial restoration of the chiral symmetry at small distances which appears as a suppression of the chiral kinetic term in (11) due to a deviation of the function $\rho(x)$ from its asymptotic value equal to 1. (See Fig.2).

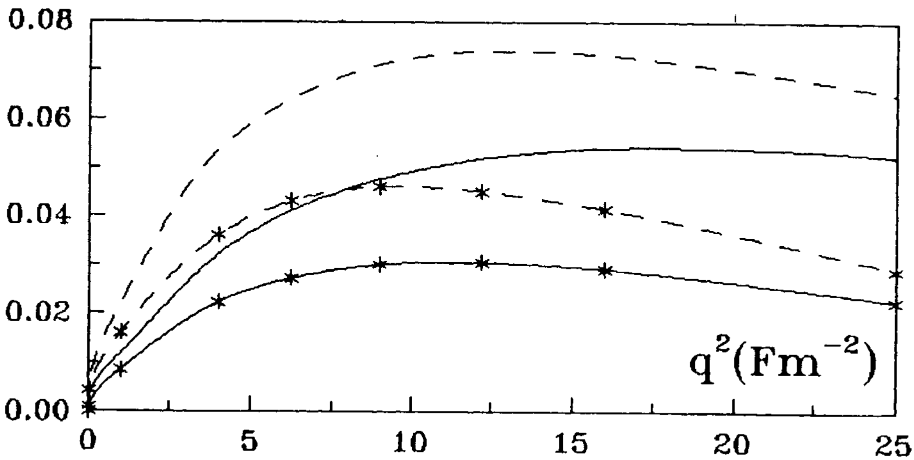

Fig.4. Electromagnetic form factor of a neutron in GSM with vibrations (solid line with centered symbols), without vibrations (dashed line with centered symbols) and OSM with vibrations (solid line), without vibrations (dashed line).

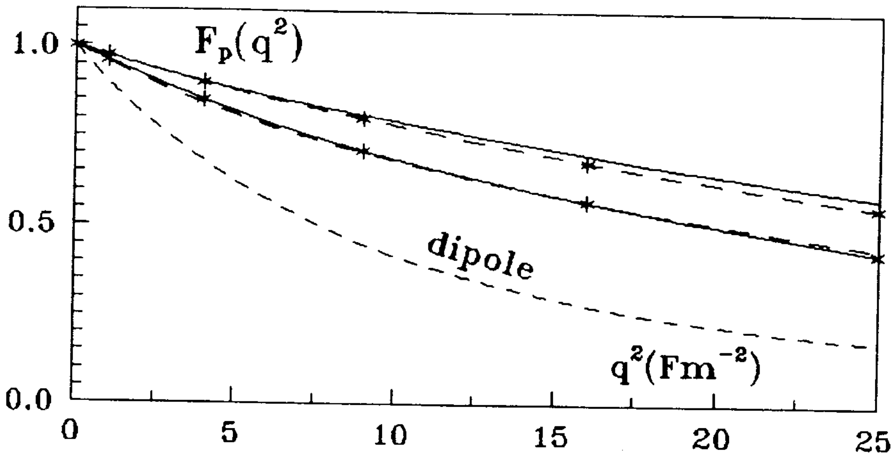

Fig.5. Electromagnetic form factor of proton in GSM and OSM. Notations are the same as for Fig.4.

The model that we have presented is based solely on chiral and conformal anomalies of the QCD and in comparison with the original Skyrme model it leads to the following results:
(a) The chiral symmetry at small distances partially restored due to the suppression of the chiral kinetic term;
(b) The classical Skyrmion mass crucially decreases;
(c) Skyrmion is a very compact object that leads to a large value of the $N - \Delta$ mass splitting;
(d) The nucleon mass in this model is in good agreement with the experimental one.

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